



Questions

1 We toss a fair coin (without bias) 2022 times and on each one of them the coin lands on heads.

What is the probability that if we toss the coin again, we will obtain tails?

2 A natural number n is “multiplicatively perfect” if the product of all of its divisors equals n^2 (equivalently, either $n=1$ or the product of its proper divisors equals n).

What is the 35th multiplicatively perfect number?

¿Cuál es el trigésimo quinto número multiplicativamente perfecto?

3 Given a vector V with values on $\mathbb{Z}/5\mathbb{Z}$, the “pattern” of V is the sequence of its nonzero entries. For example, the pattern of $(0,3,2,0,1)$ is $(3,2,1)$, and the pattern of $(0,0,2,0,4,0,0,0)$ is $(2,4)$.

We will say that a pattern is “additive” if there are two vectors (with n entries, for some n) x, y such that x, y and $x+y$ all have the same pattern.

What is the size of the smallest additive pattern in $\mathbb{Z}/5\mathbb{Z}$?

4 Consider the following puzzle: There are three rods arranged in a line and five disks of different sizes with holes so they can be slid into the rods. Let us denote the disks, from smaller to larger, by T, S, M, L, H . The disks can be freely moved and stacked on the rods, subject to the following rules:

Each disk can only be on top of a larger disk.

Only one disk can be moved at each step.

Only the disk at the top of each rod’s stack can be moved.

Each move has a positive cost: it costs more to move a disk between non-adjacent rods than to move it between adjacent rods, and it costs more to move larger disks than to move smaller ones, no matter how far the move is. The initial state of the puzzle is that of



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five disks in a single rod: in order from bottom to top, H,L,M,S,T, If we encode moves by disk names, what is the sequence of moves with least cost such that all five disks are in a single rod, different from the initial one?

5 Solve the following maze with a minimal length path, starting from the cyan colored square and exists marked by and **||**.

Write your solutions indicating movement directions at each step, as viewed from the top. Available movement directions are

L- left R - right U - up D - down

Example: The path from the cyan colored square to the exit is RRRD

